Chapter 0 Functions

0.1 Functions and Their Graphs

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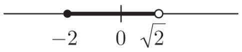
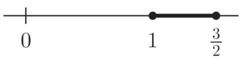
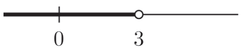
3. 

4. 

5. 

6. 

7. [2, 3)

8. $\left(-1, \frac{3}{2}\right)$

9. [–1, 0)

10. [–1, 8)

11. $(-\infty, 3)$

12. $\left[\sqrt{2}, \infty\right)$

13. $f(x) = x^2 - 3x$

$f(0) = 0^2 - 3(0) = 0$

$f(5) = 5^2 - 3(5) = 25 - 15 = 10$

$f(3) = 3^2 - 3(3) = 9 - 9 = 0$

$f(-7) = (-7)^2 - 3(-7) = 49 + 21 = 70$

14. $f(x) = x^3 + x^2 - x - 1$

$f(1) = 1^3 + 1^2 - 1 - 1 = 0$

$f(-1) = (-1)^3 + (-1)^2 - (-1) - 1 = 0$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -\frac{9}{8}$

$f(a) = a^3 + a^2 - a - 1$

15. $f(x) = x^2 - 2x$

$f(a+1) = (a+1)^2 - 2(a+1)$
$= (a^2 + 2a + 1) - 2a - 2 = a^2 - 1$

$f(a+2) = (a+2)^2 - 2(a+2)$
$= (a^2 + 4a + 4) - 2a - 4 = a^2 + 2a$

16. $h(s) = \frac{s}{(1+s)}$

$h\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(1+\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

$h\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}}{1+\left(-\frac{3}{2}\right)} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

$h(a+1) = \frac{a+1}{1+(a+1)} = \frac{a+1}{a+2}$

17. $f(x) = 3x + 2,\ h \neq 0$

$f(3+h) = 3(3+h) + 2 = 9 + 3h + 2 = 3h + 11$

$f(3) = 3(3) + 2 = 11$

$\frac{f(3+h) - f(3)}{h} = \frac{(3h+11) - 11}{h} = \frac{3h}{h} = 3$

18. $f(x) = x^2,\ h \neq 0$

$f(1+h) = (1+h)^2 = 1 + 2h + h^2$

$f(1) = 1^2 = 1$

$\frac{f(1+h) - f(1)}{h} = \frac{(1 + 2h + h^2) - 1}{h}$
$= \frac{2h + h^2}{h} = 2 + h$

19. **a.** $k(x) = x + 273$

$5933 = x + 273 \Rightarrow x = 5660$

The boiling point of tungsten is 5660°C.

b. $f(x) = \frac{9}{5}x + 32$

$f(x) = \frac{9}{5}(5660) + 32 = 10220$

The boiling point of tungsten is 10220°F.

20. **a.** $f(0)$ represents the number of laptops sold in 2015.

b. $f(5) = 150 + 2(5) + 5^2$
$= 150 + 10 + 25 = 185$

In 2020, the company will sell 185 laptops.

21. $f(x) = \frac{8x}{(x-1)(x-2)}$

all real numbers such that $x \neq 1, 2$ or $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

22. $f(t)=\dfrac{1}{\sqrt{t}}$

all real numbers such that $t>0$ or $(0,\infty)$

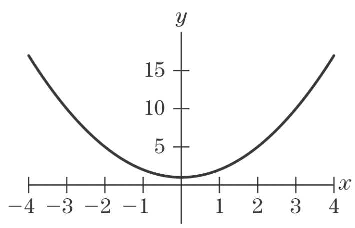
23. $g(x)=\dfrac{1}{\sqrt{3-x}}$

all real numbers such that $x<3$ or $(-\infty,-3)$

24. $g(x)=\dfrac{4}{x(x+2)}$

all real numbers such that $x\neq 0, -2$ or $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

25.



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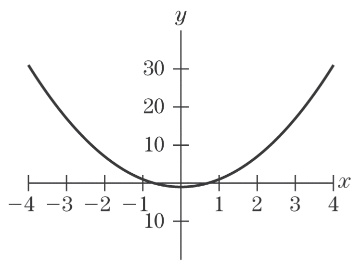

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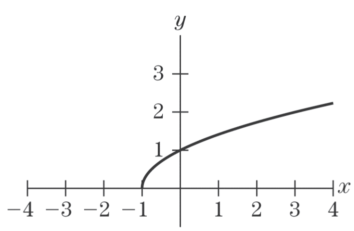

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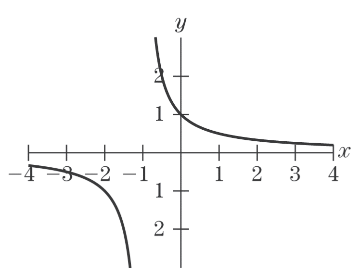


29. function **30.** not a function

31. not a function **32.** not a function

33. not a function **34.** function

35. $f(0)=1;\quad f(7)=-1$

36. $f(2)=3;\quad f(-1)=0$

37. positive **38.** negative

39. $[-1, 3]$ **40.** $-1, 5, 9$

41. $(-\infty, -1]\cup[5, 9]$ **42.** $[-1,5]\cup[9,\infty]$

43. $f(1)\approx .03;\quad f(5)\approx .037$

44. $f(6)\approx .03$

45. $[0, .05]$ **46.** $t\approx 3$

47. $f(x)=\left(x-\dfrac{1}{2}\right)(x+2)$

$f(3)=\left(3-\dfrac{1}{2}\right)(3+2)=\dfrac{25}{2}$

No, (3, 12) is not on the graph.

48. $f(x)=x(5+x)(4-x)$

$f(-2)=-2(5+(-2))(4-(-2))=-36$

No, (–2, 12) is not on the graph.

49. $g(x)=\dfrac{3x-1}{x^2+1}$

$g(1)=\dfrac{3(1)-1}{(1)^2+1}=\dfrac{2}{2}=1$

Yes, $(1, 1)$ is on the graph.

50. $g(x)=\dfrac{x^2+4}{x+2}$

$g(4)=\dfrac{(4)^2+4}{4+2}=\dfrac{20}{6}=\dfrac{10}{3}$

No, $\left(4, \dfrac{1}{4}\right)$ is not on the graph.

51. $f(x)=x^3$

$f(a+1)=(a+1)^3$

52. $f(x)=\left(\dfrac{5}{x}\right)-x$

$f(2+h)=\dfrac{5}{(2+h)}-(2+h)$

$=\dfrac{5-(2+h)^2}{(2+h)}=\dfrac{1-4h-h^2}{2+h}$

53. $f(x)=\begin{cases}\sqrt{x} & \text{for } 0\le x<2\\ 1+x & \text{for } 2\le x\le 5\end{cases}$

$f(1)=\sqrt{1}=1$

$f(2)=1+2=3$

$f(3)=1+3=4$

54. $f(x)=\begin{cases}\dfrac{1}{x} & \text{for } 1\le x\le 2\\ x^2 & \text{for } 2<x\end{cases}$

$f(1)=\dfrac{1}{1}=1$

$f(2)=\dfrac{1}{2}$

$f(3)=3^2=9$

55. $f(x)=\begin{cases}\pi x^2 & \text{for } x<2\\ 1+x & \text{for } 2\le x\le 2.5\\ 4x & \text{for } 2.5<x\end{cases}$

$f(1)=\pi(1)^2=\pi$
$f(2) = 1 + 2 = 3$
$f(3) = 4(3) = 12$

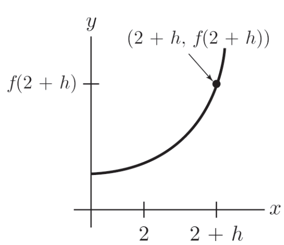
56. $f(x)=\begin{cases}\dfrac{3}{4-x} & \text{for } x<2\\ 2x & \text{for } 2\le x<3\\ \sqrt{x^2-5} & \text{for } 3\le x\end{cases}$

$f(1)=\dfrac{3}{4-1}=1$
$f(2) = 2(2) = 4$
$f(3)=\sqrt{3^2-5}=\sqrt{4}=2$

57. a. $f(x)=\begin{cases}0.06x & \text{for } 50\le x\le 3000\\ 0.02x+15 & \text{for } 3000<x\end{cases}$

b. $f(3000)=0.06(3000)=180$
$f(4500)=0.02(4500)+15=105$

58.



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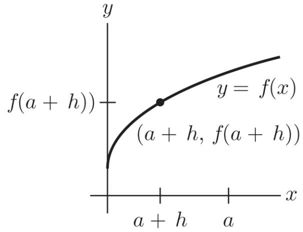


60. $R(x)=\dfrac{100x}{b+x},\ x\ge 0$

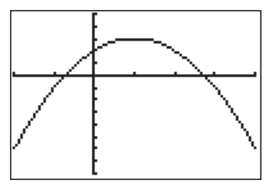
a. $R(30)=\dfrac{100(30)}{15+30}=\dfrac{3000}{45}=\dfrac{200}{3}$

b. $30=\dfrac{100(50)}{b+50}\Rightarrow b+50=\dfrac{5000}{30}\Rightarrow$
$b=\dfrac{500}{3}-50=\dfrac{350}{3}$

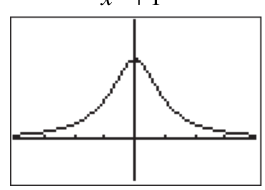
61. Entering **Y_1 = 1/X + 1** will graph the function $f(x)=\dfrac{1}{x}+1$. In order to graph the function $f(x)=\dfrac{1}{x+1}$, you need to include parentheses in the denominator: **Y_1 = 1/(X + 1)**.

62. Entering **Y_1 = X ^ 3 / 4** will graph the function $f(x)=\dfrac{x^3}{4}$. In order to graph the function $y=x^{3/4}$, you need to include parentheses in the exponent: **Y_1 = X ^ (3/4)**.

63. $f(x)=-x^2+2x+2$



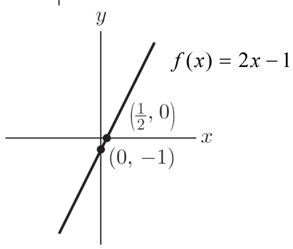
64. $f(x)=\dfrac{1}{x^2+1}$



0.2 Some Important Functions

1. $y = 2x - 1$

x	y
1	1
0	–1
–1	–3



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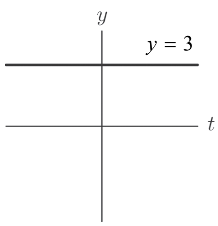
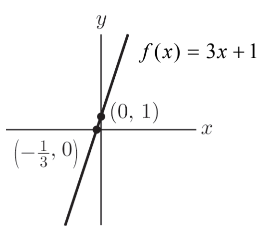


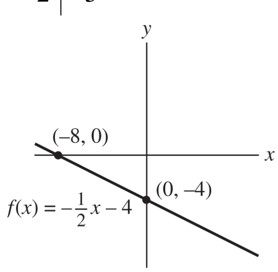
3. $y = 3x + 1$

x	y
1	4
0	1
–1	–2



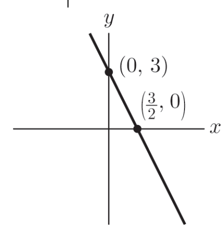
4. $y = -\dfrac{1}{2}x - 4$

x	y
2	–5
0	–4
–2	–3

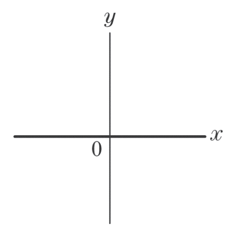


5. $y = -2x + 3$

x	y
−1	5
0	3
1	1

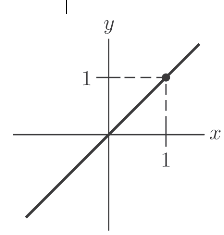


6. $y = 0$



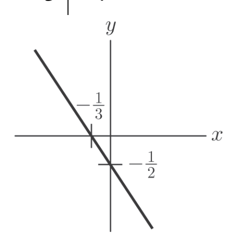
7. $x - y = 0$

x	y
1	1
0	0
–1	–1



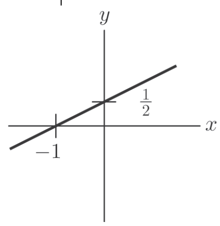
8. $3x + 2y = -1$

x	y
3	−5
1	–2
−3	4



9. $x = 2y - 1$

x	y
3	2
1	1
–3	–1



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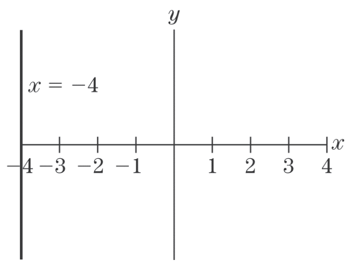


11. $f(x) = 9x + 3$
$f(0) = 9(9) + 3 = 3$
The y-intercept is (0, 3).
$9x + 3 = 0 \Rightarrow 9x = -3 \Rightarrow x = -\frac{1}{3}$
The x-intercept is $\left(-\frac{1}{3}, 0\right)$.

12. $f(x) = -\frac{1}{2}x - 1$
$f(0) = -\frac{1}{2}(0) - 1 = -1$
The y-intercept is (0, –1).
$-\frac{1}{2}x - 1 = 0 \Rightarrow -\frac{1}{2}x = 1 \Rightarrow x = -2$
The x-intercept is (–2, 0).

13. $f(x) = 5$
The y-intercept is (0, 5).
There is no x-intercept.

14. $f(x) = 14$
The y-intercept is (0, 14).
There is no x-intercept.

15. $x - 5y = 0$
$0 - 5y = 0 \Rightarrow y = 0$
The x- and y-intercept is (0, 0).

16. $2 + 3x = 2y$
$2 + 3(0) = 2y \Rightarrow y = 1$
The y-intercept is (0, 1).
$2 + 3x = 2(0) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$
The x-intercept is $\left(-\frac{2}{3}, 0\right)$.

17. a. Cost is \$(24 + 200(.45)) = \$114.

b. $f(x) = .45x + 24$

18. Let x be the volume of gas (in thousands of cubic feet) extracted.
$f(x) = 5000 + .10x$

19. Let x be the number of days of hospital confinement.
$f(x) = 700x + 1900$

20. $6x - 40 = 350 \Rightarrow x = 65$ mph

21. $f(x) = \frac{50x}{105 - x}, \ 0 \le x \le 100$
From example 6, we know that $f(70) = 100$. The cost to remove 75% of the pollutant is
$f(75) = \frac{50 \cdot 75}{105 - 75} = 125.$
The cost of removing an extra 5% is \$125 − \$100 = \$25 million. To remove the final 5% the cost is
$f(100) - f(95) = 1000 - 475 =$ \$525 million.
This costs 21 times as much as the cost to remove the next 5% after the first 70% is removed.

22. a. $f(85) = \frac{20(85)}{102 - 85} = \$100$ million

b. $f(100) - f(95) = 1000 - 271.43 \approx$ \$728.57 million

23. $f(x) = \left(\frac{K}{V}\right)x + \frac{1}{V}$

a. $f(x) = .2x + 50$
We have $\frac{K}{V} = .2$ and $\frac{1}{V} = 50.$ If $\frac{1}{V} = 50,$ then $V = \frac{1}{50}.$ Now, $\frac{K}{V} = .2$ implies $\frac{K}{\frac{1}{50}} = .2,$ so $K = \frac{1}{5} \cdot \frac{1}{50} = \frac{1}{250}.$

b. $y = \left(\frac{K}{V}\right)x + \frac{1}{V}, \left(\frac{K}{V}\right)\cdot 0 + \frac{1}{V} = \frac{1}{V}$, so the y-intercept is $\left(0, \frac{1}{V}\right)$.

Solving $\left(\frac{K}{V}\right)x + \frac{1}{V} = 0$, we get $\frac{K}{V}x = -\frac{1}{V} \Rightarrow x = -\frac{1}{K}$, so the x-intercept is $\left(-\frac{1}{K}, 0\right)$.

24. From 17(b), $\left(-\frac{1}{K}, 0\right)$ is the x-intercept. From the experimental data, (–500, 0) is also the x-intercept. Thus $-\frac{1}{K} = -500 \Rightarrow K = \frac{1}{500}$.
Again from 17(b), $\left(0, \frac{1}{V}\right)$ is the y-intercept. From the experimental data, (0, 60) is also the y-intercept. Thus $\frac{1}{V} = 60 \Rightarrow V = \frac{1}{60}$.

25. $y = 3x^2 - 4x$
$a = 3, b = -4, c = 0$

26. $y = \frac{x^2 - 6x + 2}{3} = \frac{1}{3}x^2 - 2x + \frac{2}{3}$
$a = \frac{1}{3},\ b = -2,\ c = \frac{2}{3}$

27. $y = 3x - 2x^2 + 1$
$a = -2, b = 3, c = 1$

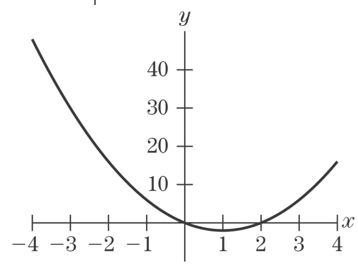
28. $y = 3 - 2x + 4x^2$
$a = 4, b = -2, c = 3$

29. $y = 1 - x^2$
$a = -1, b = 0, c = 1$

30. $y = \frac{1}{2}x^2 + \sqrt{3}\,x - \pi$
$a = \frac{1}{2},\ b = \sqrt{3},\ c = -\pi$

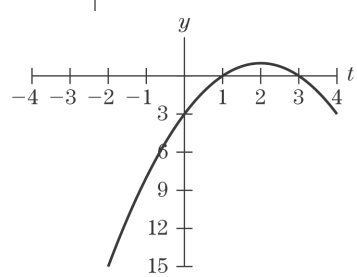
31. $f(x) = 2x^2 - 4x$
$a = 2, b = -4, c = 0$
vertex:
$\left(\frac{-(-4)}{2(2)}, f\left(\frac{-(-4)}{2(2)}\right)\right) = (1, f(1)) = (1, -2)$

x	y
0	0
2	0



32. $g(t) = -t^2 + 4t - 3$
$a = -1, b = 4, c = -3$
vertex:
$\left(\frac{-4}{2(-1)}, g\left(\frac{-4}{2(-1)}\right)\right) = (2, g(2)) = (2, 1)$

x	y
0	–3
1	0
3	0



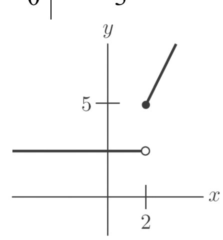
33. $f(x) = \begin{cases} 3 & \text{for } x < 2 \\ 2x + 1 & \text{for } x \ge 2 \end{cases}$

$x < 2$

x	$f(x) = 3$
1	3
0	3

$x \ge 2$

x	$f(x) = 2x + 1$
2	5
3	7



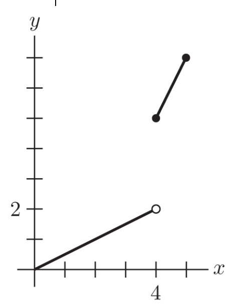
34. $f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 \le x < 4 \\ 2x - 3 & \text{for } 4 \le x \le 5 \end{cases}$

$0 \le x < 4$

x	$f(x) = \frac{1}{2}x$
0	0
2	1
3	$\frac{3}{2}$

$4 \le x \le 5$

x	$f(x) = 2x - 3$
4	5
5	7



35. $f(x) = \begin{cases} 4 - x & \text{for } 0 \le x < 2 \\ 2x - 2 & \text{for } 2 \le x < 3 \\ x + 1 & \text{for } x \ge 3 \end{cases}$

$0 \le x < 2$

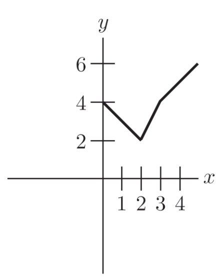
x	$f(x) = 4 - x$
0	4
1	3

$2 \le x < 3$

x	$f(x) = 2x - 2$
2	2
$\frac{5}{2}$	3

$x \ge 3$

x	$f(x) = x + 1$
3	4
4	5



36. $f(x) = \begin{cases} 4x & \text{for } 0 \le x < 1 \\ 8 - 4x & \text{for } 1 \le x < 2 \\ 2x - 4 & \text{for } x \ge 2 \end{cases}$

$0 \le x < 1$

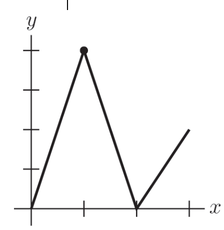
x	$f(x) = 4x$
0	0
$\frac{1}{2}$	2

$1 \le x < 2$

x	$f(x) = 8 - 4x$
1	4
$\frac{3}{2}$	2

$x \ge 2$

x	$f(x) = 2x - 4$
2	0
3	2



37. $f(x) = x^{100},\ x = -1$
$f(-1) = (-1)^{100} = 1$

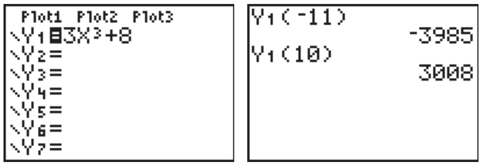
38. $f(x) = x^5,\ x = \frac{1}{2}$
$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

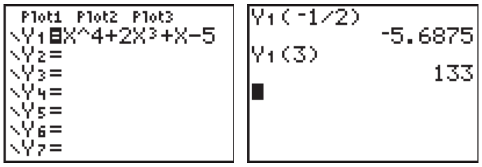
39. $f(x) = |x|,\ x = 10^{-2}$
$f(10^{-2}) = |10^{-2}| = 10^{-2}$

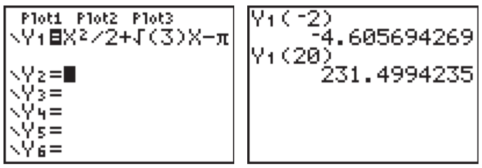
40. $f(x) = |x|,\ x = \pi$
$f(\pi) = |\pi| = \pi$

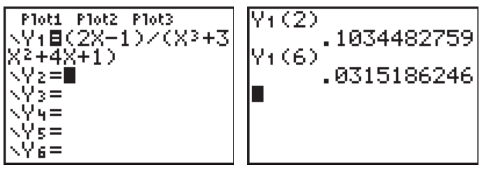
41. $f(x) = |x|,\ x = -2.5$
$f(-2.5) = |-2.5| = 2.5$

42. $f(x) = |x|,\ x = -\frac{2}{3}$
$f\left(-\frac{2}{3}\right) = \left|-\frac{2}{3}\right| = \frac{2}{3}$

43. 

44. 

45. 

46. 

0.3 The Algebra of Functions

1. $f(x)+g(x)=(x^2+1)+9x=x^2+9x+1$

2. $f(x)-h(x)=(x^2+1)-(5-2x^2)=3x^2-4$

3. $f(x)g(x)=(x^2+1)(9x)=9x^3+9x$

4. $g(x)h(x)=(9x)(5-2x^2)=45x-18x^3$

5. $\dfrac{f(t)}{g(t)}=\dfrac{t^2+1}{9t}=\dfrac{t^2}{9t}+\dfrac{1}{9t}=\dfrac{t}{9}+\dfrac{1}{9t}=\dfrac{t^2+1}{9t}$

6. $\dfrac{g(t)}{h(t)}=\dfrac{9t}{5-2t^2}$

7. $\dfrac{2}{x-3}+\dfrac{1}{x+2}=\dfrac{2(x+2)+(x-3)}{(x-3)(x+2)}=\dfrac{3x+1}{x^2-x-6}$

8. $\dfrac{3}{x-6}+\dfrac{-2}{x-2}=\dfrac{3(x-2)+(-2)(x-6)}{(x-6)(x-2)}=\dfrac{x+6}{x^2-8x+12}$

9. $\dfrac{x}{x-8}+\dfrac{-x}{x-4}=\dfrac{x(x-4)+(-x)(x-8)}{(x-8)(x-4)}=\dfrac{4x}{x^2-12x+32}$

10. $\dfrac{-x}{x+3}+\dfrac{x}{x+5}=\dfrac{(-x)(x+5)+x(x+3)}{(x+3)(x+5)}=\dfrac{-2x}{x^2+8x+15}$

11. $\dfrac{x+5}{x-10}+\dfrac{x}{x+10}=\dfrac{(x+5)(x+10)+x(x-10)}{(x-10)(x+10)}=\dfrac{2x^2+5x+50}{x^2-100}$

12. $\dfrac{x+6}{x-6}+\dfrac{x-6}{x+6}=\dfrac{(x+6)(x+6)+(x-6)(x-6)}{(x-6)(x+6)}=\dfrac{2x^2+72}{x^2-36}$

13. $\dfrac{x}{x-2}-\dfrac{5-x}{5+x}=\dfrac{x(5+x)-(5-x)(x-2)}{(x-2)(5+x)}=\dfrac{2x^2-2x+10}{x^2+3x-10}$

14. $\dfrac{t}{t-2}-\dfrac{t+1}{3t-1}=\dfrac{t(3t-1)-(t-2)(t+1)}{(t-2)(3t-1)}=\dfrac{2t^2+2}{3t^2-7t+2}$

15. $\dfrac{x}{x-2}\cdot\dfrac{5-x}{5+x}=\dfrac{-x^2+5x}{x^2+3x-10}$

16. $\dfrac{5-x}{5+x}\cdot\dfrac{x+1}{3x-1}=\dfrac{-x^2+4x+5}{3x^2+14x-5}$

17. $\dfrac{\frac{x}{x-2}}{\frac{5-x}{5+x}}=\dfrac{x}{x-2}\cdot\dfrac{5+x}{5-x}=\dfrac{x^2+5x}{-x^2+7x-10}$

18. $\dfrac{\frac{s+1}{3s-1}}{\frac{s}{s-2}}=\dfrac{s+1}{3s-1}\cdot\dfrac{s-2}{s}=\dfrac{s^2-s-2}{3s^2-s}$

19. $\dfrac{x+1}{(x+1)-2}\cdot\dfrac{5-(x+1)}{5+(x+1)}=\dfrac{x+1}{x-1}\cdot\dfrac{-x+4}{6+x}=\dfrac{-x^2+3x+4}{x^2+5x-6}$

20. $\dfrac{x+2}{(x+2)-2}+\dfrac{5-(x+2)}{5+(x+2)}=\dfrac{x+2}{x}+\dfrac{3-x}{x+7}=\dfrac{(x+2)(x+7)+(3-x)(x)}{x(x+7)}=\dfrac{12x+14}{x^2+7x}$

21. $\dfrac{\dfrac{5-(x+5)}{5+(x+5)}}{\dfrac{x+5}{(x+5)-2}} = \dfrac{5-(x+5)}{5+(x+5)} \cdot \dfrac{(x+5)-2}{x+5}$

$= \dfrac{-x}{10+x} \cdot \dfrac{x+3}{x+5}$

$= \dfrac{-x^2-3x}{x^2+15x+50}$

22. $\dfrac{\frac{1}{t}}{\frac{1}{t}-2} = \dfrac{1}{t} \cdot \dfrac{t}{1-2t} = \dfrac{1}{1-2t}$, $t \neq 0$

23. $\dfrac{5-\frac{1}{u}}{5+\frac{1}{u}} = \dfrac{5u-1}{u} \cdot \dfrac{u}{5u+1} = \dfrac{5u-1}{5u+1}$, $u \neq 0$

24. $\dfrac{\frac{1}{x^2}+1}{3\left(\frac{1}{x^2}\right)-1} = \dfrac{1+x^2}{x^2} \cdot \dfrac{x^2}{3-x^2} = \dfrac{1+x^2}{3-x^2}$, $x \neq 0$

25. $f\left(\dfrac{x}{1-x}\right) = \left(\dfrac{x}{1-x}\right)^6$

26. $h\left(t^6\right) = \left(t^6\right)^3 - 5\left(t^6\right)^2 + 1 = t^{18} - 5t^{12} + 1$

27. $h\left(\dfrac{x}{1-x}\right) = \left(\dfrac{x}{1-x}\right)^3 - 5\left(\dfrac{x}{1-x}\right)^2 + 1$

28. $g\left(x^6\right) = \dfrac{x^6}{1-x^6}$

29. $g(t^3-5t^2+1) = \dfrac{t^3-5t^2+1}{1-(t^3-5t^2+1)}$

$= \dfrac{t^3-5t^2+1}{-t^3+5t^2}$

30. $f(x^3-5x^2+1) = (x^3-5x^2+1)^6$

31. $(x+h)^2 - x^2 = x^2 + 2xh + h^2 - x^2$

$= 2xh + h^2$

32. $\dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{x-x-h}{x(x+h)} = \dfrac{-h}{x^2+xh}$

33. $\dfrac{\left[4(t+h)-(t+h)^2\right]-\left(4t-t^2\right)}{h}$

$= \dfrac{4t+4h-(t^2+2th+h^2)-4t+t^2}{h}$

$= \dfrac{4h-2th-h^2}{h} = \dfrac{h(4-2t-h)}{h}$

$= 4-2t-h$

34. $\dfrac{\left[(t+h)^3+5\right]-\left(t^3+5\right)}{h}$

$= \dfrac{t^3+3t^2h+3th^2+h^3+5-t^3-5}{h}$

$= \dfrac{3t^2h+3th^2+h^3}{h} = \dfrac{h(3t^2+3th+h^2)}{h}$

$= 3t^2+3th+h^2$

35. **a.** $C(A(t)) = 3000 + 80\left(20t - \dfrac{1}{2}t^2\right)$

$= 3000 + 1600t - 40t^2$

b. $C(2) = 3000 + 1600(2) - 40(2)^2$

$= 3000 + 3200 - 160 = \$6040$

36. **a.** $C(f(t))$

$= .1(10t-5)^2 + 25(10t-5) + 200$

$= .1(100t^2 - 100t + 25) + 250t - 125 + 200$

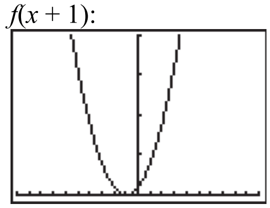
$= 10t^2 + 240t + 77.5$

b. $C(4) = 10(4)^2 + 240(4) + 77.5 = \$1197.50$

37. $h(x) = f(8x+1) = \left(\dfrac{1}{8}\right)(8x+1) = x + \dfrac{1}{8}$

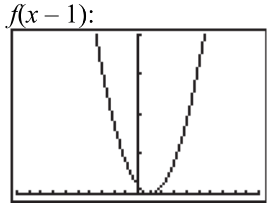
$h(x)$ converts from British to U.S. sizes.

38. $f(x+1)$:



[−10, 10] by [0, 20]

$f(x-1)$:

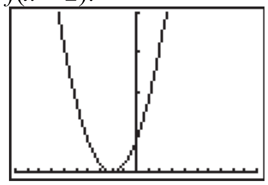


[−10, 10] by [0, 20]

(*continued on next page*)

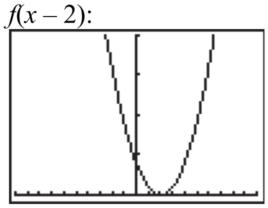
(*continued*)

$f(x+2)$:



[−10, 10] by [0, 20]

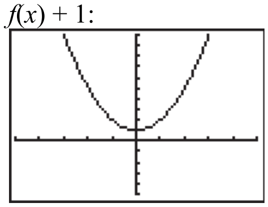
$f(x-2)$:



[−10, 10] by [0, 20]

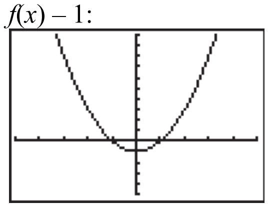
The graph of $f(x+a)$ is the graph of $f(x)$ shifted to the left (if $a > 0$) or to the right (if $a < 0$) by $|a|$ units.

39. $f(x)+1$:



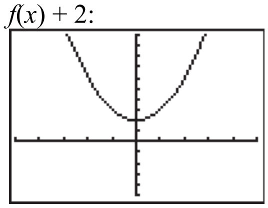
[−5, 5] by [−5, 15]

$f(x)-1$:



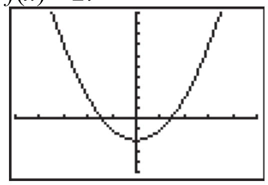
[−5, 5] by [−5, 15]

$f(x)+2$:



[−5, 5] by [−5, 15]

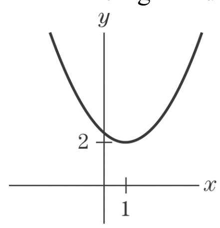
$f(x)-2$:

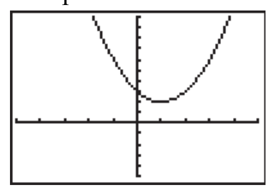


[−5, 5] by [−5, 15]

The graph of $f(x)+c$ is the graph of $f(x)$ shifted up (if $c > 0$) or down (if $c < 0$) by $|c|$ units.

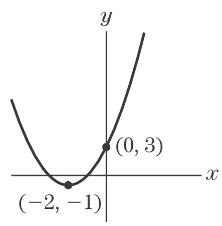
40. This is the graph of $f(x) = x^2$ shifted 1 unit to the right and 2 units up.

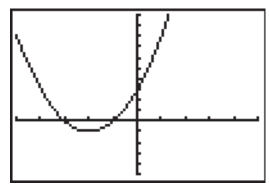




[−5, 5] by [−5, 15]

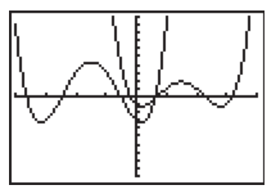
41. This is the graph of $f(x) = x^2$ shifted 2 units to the left and 1 unit down.





[−5, 5] by [−5, 15]

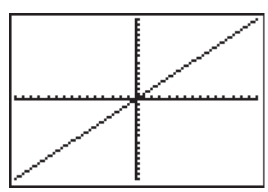
42.



[−4, 4] by [−10, 10]

They are not the same function.

43.



[−15, 15] by [−10, 10]

$$f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$$

$$= \frac{x}{x-(x-1)} = x,\ x \neq 1$$

0.4 Zeros of Functions—The Quadratic Formula and Factoring

1. $f(x)=2x^2-7x+6$

$2x^2-7x+6=0$

$a=2,\ b=-7,\ c=6$

$\sqrt{b^2-4ac}=\sqrt{49-4(2)(6)}=\sqrt{1}=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{7\pm1}{4}=2,\ \frac{3}{2}$$

2. $f(x)=3x^2+2x-1$

$3x^2+2x-1=0$

$a=3,\ b=2,\ c=-1$

$\sqrt{b^2-4ac}=\sqrt{4^2-4(3)(-1)}=\sqrt{16}=4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm4}{6}=\frac{1}{3},\ -1$$

3. $f(t)=4t^2-12t+9$

$4t^2-12t+9=0$

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{12\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)}$$

$$=\frac{12\pm\sqrt{0}}{8}=\frac{3}{2}$$

4. $f(x)=\frac{1}{4}x^2+x+1$

$\frac{1}{4}x^2+x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4\left(\frac{1}{4}\right)(1)}}{2\left(\frac{1}{4}\right)}$$

$$=\frac{-1\pm\sqrt{0}}{\frac{1}{2}}=-2$$

5. $f(x)=-2x^2+3x-4$

$-2x^2+3x-4=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{3^2-4(-2)(-4)}}{2(-2)}$$

$$=\frac{-3\pm\sqrt{-23}}{-4}$$

$\sqrt{-23}$ is undefined, so $f(x)$ has no real zeros.

6. $f(a)=11a^2-7a+1$

$11a^2-7a+1=0$

$$a=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{7\pm\sqrt{(-7)^2-4(11)(1)}}{2(11)}$$

$$=\frac{7\pm\sqrt{5}}{22}=\frac{7+\sqrt{5}}{22},\ \frac{7-\sqrt{5}}{22}$$

7. $5x^2-4x-1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{(-4)^2-4(5)(-1)}}{2(5)}$$

$$=\frac{4\pm\sqrt{36}}{10}=\frac{4\pm6}{10}=1,\ -\frac{1}{5}$$

8. $x^2-4x+5=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$$

$$=\frac{4\pm\sqrt{-4}}{2}$$

$\sqrt{-4}$ is undefined, so there is no real solution.

9. $15x^2-135x+300=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{135\pm\sqrt{(-135)^2-4(15)(300)}}{2(15)}$$

$$=\frac{135\pm\sqrt{225}}{30}=\frac{135\pm15}{30}=5,\ 4$$

10. $z^2-\sqrt{2}\,z-\frac{5}{4}=0$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{\sqrt{2}\pm\sqrt{\left(-\sqrt{2}\right)^2-4(1)\left(-\frac{5}{4}\right)}}{2(1)}=\frac{\sqrt{2}\pm\sqrt{7}}{2}$$

$$=\frac{\sqrt{2}+\sqrt{7}}{2},\ \frac{\sqrt{2}-\sqrt{7}}{2}$$

11. $\frac{3}{2}x^2-6x+5=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{6\pm\sqrt{(-6)^2-4\left(\frac{3}{2}\right)(5)}}{2\left(\frac{3}{2}\right)}$$

$$=\frac{6\pm\sqrt{6}}{3}=2+\frac{\sqrt{6}}{3},\ 2-\frac{\sqrt{6}}{3}$$

12. $9x^2 - 12x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$$
$$= \frac{12 \pm \sqrt{0}}{18} = \frac{2}{3}$$

13. $x^2 + 8x + 15 = (x + 5)(x + 3)$

14. $x^2 - 10x + 16 = (x - 2)(x - 8)$

15. $x^2 - 16 = (x - 4)(x + 4)$

16. $x^2 - 1 = (x + 1)(x - 1)$

17. $3x^2 + 12x + 12 = 3\left(x^2 + 4x + 4\right)$
$= 3(x + 2)(x + 2) = 3(x + 2)^2$

18. $2x^2 - 12x + 18 = 2\left(x^2 - 6x + 9\right)$
$= 2(x - 3)(x - 3) = 2(x - 3)^2$

19. $30 - 4x - 2x^2 = -2\left(-15 + 2x + x^2\right)$
$= -2(x - 3)(x + 5)$

20. $15 + 12x - 3x^2 = -3(-5 - 4x + x^2)$
$= -3(x - 5)(x + 1)$

21. $3x - x^2 = x(3 - x)$

22. $4x^2 - 1 = (2x + 1)(2x - 1)$

23. $6x - 2x^3 = -2x(x^2 - 3)$
$= -2x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$

24. $16x + 6x^2 - x^3 = x\left(16 + 6x - x^2\right)$
$= x(8 - x)(x + 2)$
$= -x(x - 8)(x + 2)$

25. $x^3 - 1 = (x - 1)\left(x^2 + x + 1\right)$

26. $x^3 + 125 = (x + 5)\left(x^2 - 5x + 25\right)$

27. $8x^3 + 27 = (2x + 3)\left(4x^2 - 6x + 9\right)$

28. $x^3 - \frac{1}{8} = \left(x - \frac{1}{2}\right)\left(x^2 + \frac{x}{2} + \frac{1}{4}\right)$

29. $x^2 - 14x + 49 = (x - 7)^2$

30. $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

31. $2x^2 - 5x - 6 = 3x + 4$
$2x^2 - 8x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{144}}{4} = \frac{8 \pm 12}{4} = 5, -1$$

$y = 3x + 4 = 15 + 4 = 19$
$y = -3 + 4 = 1$
Points of intersection: (5, 19), (–1, 1)

32. $x^2 - 10x + 9 = x - 9$
$x^2 - 11x + 18 = 0$
$(x - 9)(x - 2) = 0$
$x = 9, 2$
$y = x - 9 = 9 - 9 = 0$
$y = 2 - 9 = -7$
Points of intersection: (9, 0), (2, –7)

33. $y = x^2 - 4x + 4$
$y = 12 + 2x - x^2$
$x^2 - 4x + 4 = 12 + 2x - x^2$
$2x^2 - 6x - 8 = 0$
$2(x^2 - 3x - 4) = 0$
$2(x - 4)(x + 1) = 0$
$x = 4, -1$
$y = x^2 - 4x + 4 = 4^2 - 4(4) + 4 = 4$
$y = (-1)^2 - 4(-1) + 4 = 9$
Points of intersection: (4, 4), (–1, 9)

34. $y = 3x^2 + 9$
$y = 2x^2 - 5x + 3$
$3x^2 + 9 = 2x^2 - 5x + 3$
$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3, -2$
$y = 3x^2 + 9 = 3(-3)^2 + 9 = 36$
$y = 3(-2)^2 + 9 = 21$
Points of intersection: (–3, 36), (–2, 21)

35. $y = x^3 - 3x^2 + x$

$y = x^2 - 3x$

$x^3 - 3x^2 + x = x^2 - 3x$

$x^3 - 4x^2 + 4x = 0$

$x(x^2 - 4x + 4) = 0$

$x(x-2)(x-2) = 0 \Rightarrow x = 0,\ 2$

$y = x^2 - 3x = 0^2 - 3(0) = 0$

$y = 2^2 - 3(2) = 4 - 6 = -2$

Points of intersection: (0, 0), (2, –2)

36. $y = \frac{1}{2}x^3 - 2x^2$

$y = 2x$

$\frac{1}{2}x^3 - 2x^2 = 2x$

$\frac{1}{2}x^3 - 2x^2 - 2x = 0$

$x\left(\frac{1}{2}x^2 - 2x - 2\right) = 0$

$x = 0$ or $\frac{1}{2}x^2 - 2x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4\left(\frac{1}{2}\right)(-2)}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{2 \pm \sqrt{8}}{1} = 2 + 2\sqrt{2},\ 2 - 2\sqrt{2}$$

$y = 2x = 2(0) = 0$

$y = 2\left(2 + 2\sqrt{2}\right) = 4 + 4\sqrt{2}$

$y = 2\left(2 - 2\sqrt{2}\right) = 4 - 4\sqrt{2}$

Points of intersection: (0, 0), $\left(2 + 2\sqrt{2},\ 4 + 4\sqrt{2}\right)$, $\left(2 - 2\sqrt{2},\ 4 - 4\sqrt{2}\right)$

37. $y = \frac{1}{2}x^3 + x^2 + 5$

$y = 3x^2 - \frac{1}{2}x + 5$

$\frac{1}{2}x^3 + x^2 + 5 = 3x^2 - \frac{1}{2}x + 5$

$\frac{1}{2}x^3 - 2x^2 + \frac{1}{2}x = 0$

$x\left(\frac{1}{2}x^2 - 2x + \frac{1}{2}\right) = 0$

$x = 0$ or $\frac{1}{2}x^2 - 2x + \frac{1}{2} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}}{2\left(\frac{1}{2}\right)}$$

$$= 2 \pm \sqrt{3}$$

$y = 3x^2 - \frac{1}{2}x + 5 = 3(0)^2 - \frac{1}{2}(0) + 5 = 5$

$y = 3\left(2 + \sqrt{3}\right)^2 - \frac{1}{2}\left(2 + \sqrt{3}\right) + 5 = 25 + \frac{23\sqrt{3}}{2}$

$y = 3\left(2 - \sqrt{3}\right)^2 - \frac{1}{2}\left(2 - \sqrt{3}\right) + 5 = 25 - \frac{23\sqrt{3}}{2}$

Points of intersection: (0, 5), $\left(2 - \sqrt{3},\ 25 - \frac{23\sqrt{3}}{2}\right)$, $\left(2 + \sqrt{3},\ 25 + \frac{23\sqrt{3}}{2}\right)$

38. $y = 30x^3 - 3x^2$

$y = 16x^3 + 25x^2$

$30x^3 - 3x^2 = 16x^3 + 25x^2$

$14x^3 - 28x^2 = 0$

$14x^2(x-2) = 0$

$x = 0$ or $x = 2$

$y = 30(0)^3 - 3(0)^2 = 0$

$y = 30(2)^3 - 3(2)^2 = 30(8) - 3(4) = 228$

Points of intersection: (0, 0), (2, 228)

39. $\frac{21}{x} - x = 4$

$21 - x^2 = 4x$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0 \Rightarrow x = -7,\ 3$

40. $x + \frac{2}{x-6} = 3$

$x^2 - 6x + 2 = 3x - 18$

$x^2 - 9x + 20 = 0$

$(x-4)(x-5) = 0 \Rightarrow x = 4,\ 5$

41. $x + \frac{14}{x+4} = 5$

$x^2 + 4x + 14 = 5x + 20$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3,\ -2$

42. $1 = \frac{5}{x} + \frac{6}{x^2}$

$1 = \frac{5x + 6}{x^2}$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0 \Rightarrow x = 6,\ -1$

43. $\dfrac{x^2+14x+49}{x^2+1}=0$

$x^2+14x+49=0$

$(x+7)^2=0 \Rightarrow x=-7$

44. $\dfrac{x^2-8x+16}{1+\sqrt{x}}=0$

$x^2-8x+16=0$

$(x-4)^2=0 \Rightarrow x=4$

45. $C(x) = 275 + 12x$

$R(x)=32x-.21x^2$

$C(x)=R(x)$

$275+12x=32x-.21x^2$

$.21x^2-20x+275=0$

Thus

$$x=\frac{20\pm\sqrt{(-20)^2-4(.21)275}}{.42}$$

$=16,667$ or $78,571$ subscribers

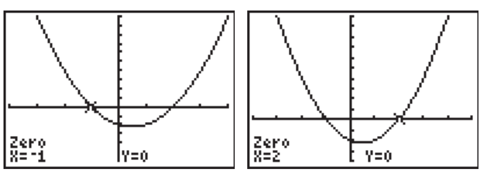
46. $x+\left(\dfrac{1}{20}\right)x^2=175$

$x^2+20x-3500=0$

$(x-50)(x+70)=0$

$x=50$ mph

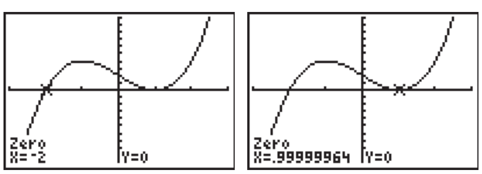
47.



[−4, 5] by [−4, 10]

The zeros are –1 and 2.

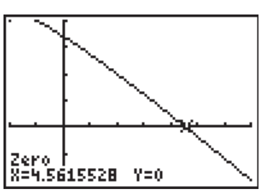
48.



[−4, 5] by [−4, 10]

The zeros are –2 and 1.

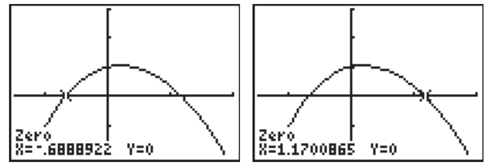
49.



[−2, 7] by [−2, 4]

The zero is approximately 4.56.

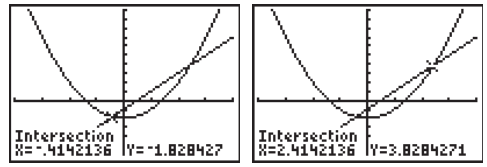
50.



[−1.5, 2] by [−2, 3]

The zeros are approximately −.689 and 1.170.

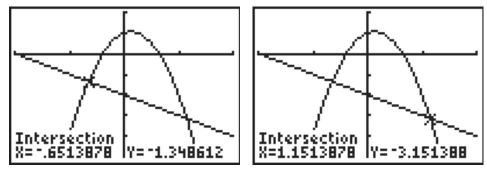
51.



[−4, 4] by [−6, 10]

Approximate points of intersection:
(–0.41, –1.83) and (2.41, 3.83)

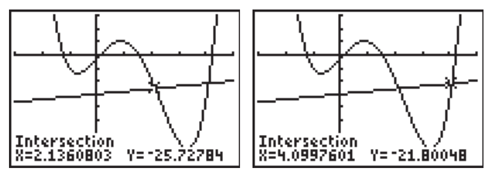
52.



[−2, 2] by [−5, 2]

Approximate points of intersection:
(–.65, –1.35) and (1.15, –3.15)

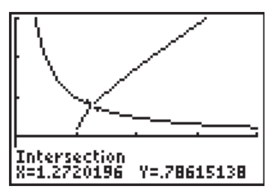
53.



[−3, 5] by [−80, 30]

Approximate points of intersection:
(2.14, –25.73) and (4.10, –21.80)

54.

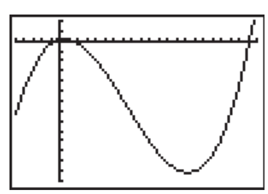


[0, 4] by [−1, 3]

Approximate point of intersection: (1.27, .79)

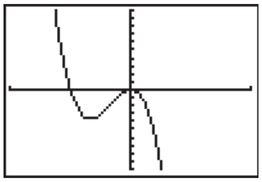
Answers may vary for exercises 55−58.

55.



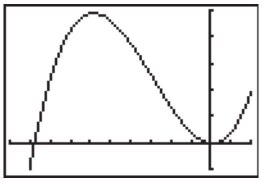
[−5, 22] by [−1400, 100]

56.



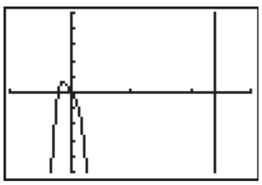
[−1, 1] by [−10, 10]

57.



[−20, 4] by [−500, 2500]

58.



[−5, 15] by [−100, 100]

0.5 Exponents and Power Functions

1. $3^3 = 27$

2. $(-2)^3 = -8$

3. $1^{100} = 1$

4. $0^{25} = 0$

5. $(.1)^4 = (.1)(.1)(.1)(.1) = .0001$

6. $(100)^4 = (100)(100)(100)(100) = 100,000,000$

7. $-4^2 = -16$

8. $(.01)^3 = .000001$

9. $(16)^{1/2} = \sqrt{16} = 4$

10. $(27)^{1/3} = \sqrt[3]{27} = 3$

11. $(.000001)^{1/3} = \sqrt[3]{.000001} = .01$

12. $\left(\frac{1}{125}\right)^{1/3} = \sqrt[3]{\frac{1}{125}} = \frac{1}{5}$

13. $6^{-1} = \frac{1}{6}$

14. $\left(\frac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = 2$

15. $(.01)^{-1} = \frac{1}{.01} = 100$

16. $(-5)^{-1} = -\frac{1}{5}$

17. $8^{4/3} = \left(\sqrt[3]{8}\right)^4 = 16$

18. $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 8$

19. $(25)^{3/2} = \left(\sqrt{25}\right)^3 = 125$

20. $(27)^{2/3} = \left(\sqrt[3]{27}\right)^2 = 9$

21. $(1.8)^0 = 1$

22. $9^{1.5} = 9^{3/2} = \left(\sqrt{9}\right)^3 = 27$

23. $16^{0.5} = 16^{1/2} = 4$

24. $81^{0.75} = 81^{3/4} = 27$

25. $4^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

26. $\left(\frac{1}{8}\right)^{-2/3} = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

27. $(.01)^{-1.5} = \frac{1}{(.01)^{3/2}} = \frac{1}{.001} = 1000$

28. $1^{-1.2} = \frac{1}{1^{1.2}} = 1$

29. $5^{1/3} \cdot 200^{1/3} = 1000^{1/3} = 10$

30. $(3^{1/3} \cdot 3^{1/6})^6 = (3^{1/2})^6 = 27$

31. $6^{1/3} \cdot 6^{2/3} = 6^1 = 6$

32. $(9^{4/5})^{5/8} = 9^{1/2} = 3$

33. $\frac{10^4}{5^4} = 2^4 = 16$

34. $\frac{3^{5/2}}{3^{1/2}} = 3^{(5/2)-(1/2)} = 3^{4/2} = 9$

35. $(2^{1/3} \cdot 3^{2/3})^3 = \left(\sqrt[3]{2}\sqrt[3]{9}\right)^3 = \left(\sqrt[3]{18}\right)^3 = 18$

36. $20^{0.5} \cdot 5^{0.5} = (100)^{1/2} = 10$

37. $\left(\frac{8}{27}\right)^{2/3} = \frac{8^{2/3}}{27^{2/3}} = \frac{4}{9}$

38. $(125 \cdot 27)^{1/3} = 125^{1/3} \cdot 27^{1/3} = 15$

39. $\frac{7^{4/3}}{7^{1/3}} = 7^{(4/3)-(1/3)} = 7^{3/3} = 7$

40. $(6^{1/2})^0 = 6^{(1/2)(0)} = 6^0 = 1$

41. $(xy)^6 = x^6 y^6$

42. $(x^{1/3})^6 = x^{(1/3)(6)} = x^2$

43. $\dfrac{x^4 \cdot y^5}{xy^2} = x^4 \cdot y^5 \cdot x^{-1} \cdot y^{-2} = x^3 y^3$

44. $\dfrac{1}{x^{-3}} = x^3$

45. $x^{-1/2} = \dfrac{1}{\sqrt{x}}$

46. $(x^3 \cdot y^6)^{1/3} = x^{3(1/3)} \cdot y^{6(1/3)} = xy^2$

47. $\left(\dfrac{x^4}{y^2}\right)^3 = \dfrac{x^{4(3)}}{y^{2(3)}} = \dfrac{x^{12}}{y^6}$

48. $\left(\dfrac{x}{y}\right)^{-2} = \dfrac{1}{x^2} \cdot y^2 = \dfrac{y^2}{x^2}$

49. $(x^3 y^5)^4 = x^{3(4)} \cdot y^{5(4)} = x^{12} y^{20}$

50. $\sqrt{1+x}(1+x)^{3/2} = (1+x)^{1/2}(1+x)^{3/2}$
$= (1+x)^{(1/2)+(3/2)} = (1+x)^2$
$= x^2 + 2x + 1$

51. $x^5 \cdot \left(\dfrac{y^2}{x}\right)^3 = \dfrac{x^5 \cdot y^{2(3)}}{x^3} = x^5 \cdot y^6 \cdot x^{-3} = x^2 y^6$

52. $x^{-3} \cdot x^7 = x^{7-3} = x^4$

53. $(2x)^4 = 2^4 \cdot x^4 = 16x^4$

54. $\dfrac{-3x}{15x^4} = -\dfrac{3}{15} \cdot \dfrac{x}{x^4} = -\dfrac{1}{5x^3}$

55. $\dfrac{-x^3 y}{-xy} = \dfrac{x^3}{x} \cdot \dfrac{y}{y} = x^2$

56. $\dfrac{x^3}{y^{-2}} = x^3 y^2$

57. $\dfrac{x^{-4}}{x^3} = \dfrac{1}{x^4} \cdot \dfrac{1}{x^3} = (-3)^3 \cdot x^3 = \dfrac{1}{x^7}$

58. $(-3x)^3 = -27x^3$

59. $\sqrt[3]{x} \cdot \sqrt[3]{x^2} = x^{1/3} \cdot x^{2/3} = x$

60. $(9x)^{-1/2} = \dfrac{1}{\sqrt{9x}} = \dfrac{1}{3\sqrt{x}}$

61. $\left(\dfrac{3x^2}{2y}\right)^3 = \dfrac{3^3 \cdot x^6}{2^3 \cdot y^3} = \dfrac{27x^6}{8y^3}$

62. $\dfrac{x^2}{x^5 y} = \dfrac{x^2}{x^5} \cdot \dfrac{1}{y} = \dfrac{1}{x^3 y}$

63. $\dfrac{2x}{\sqrt{x}} = 2x \cdot x^{-1/2} = 2\sqrt{x}$

64. $\dfrac{1}{yx^{-5}} = \dfrac{x^5}{y}$

65. $(16x^8)^{-3/4} = 16^{-3/4} \cdot x^{-6} = \dfrac{1}{8x^6}$

66. $(-8y^9)^{2/3} = (-8)^{2/3} y^{9(2/3)} = 4y^6$

67. $\sqrt{x}\left(\dfrac{1}{4x}\right)^{5/2} = \dfrac{x^{1/2}}{4^{5/2} x^{5/2}} = \dfrac{x^{1/2} \cdot x^{-5/2}}{32}$
$= \dfrac{1}{32x^2}$

68. $\dfrac{(25xy)^{3/2}}{x^2 y} = \dfrac{(25)^{3/2} x^{3/2} y^{3/2}}{x^2 y} = \dfrac{125\sqrt{y}}{\sqrt{x}}$

69. $\dfrac{(-27x^5)^{2/3}}{\sqrt[3]{x}} = \dfrac{(-27)^{2/3} x^{5(2/3)}}{x^{1/3}} = 9x^3$

70. $(-32y^{-5})^{3/5} = (-32)^{3/5} y^{-5(3/5)} = -\dfrac{8}{y^3}$

For exercises 71−82, $f(x) = \sqrt[3]{x}$ and $g(x) = \dfrac{1}{x^2}$.

71. $f(x)g(x) = \sqrt[3]{x} \cdot \dfrac{1}{x^2} = x^{1/3} \cdot x^{-2} = x^{-5/3} = \dfrac{1}{x^{5/3}}$

72. $\dfrac{f(x)}{g(x)} = \dfrac{\sqrt[3]{x}}{\frac{1}{x^2}} = x^{1/3} \cdot x^2 = x^{7/3}$

73. $\dfrac{g(x)}{f(x)} = \dfrac{\frac{1}{x^2}}{\sqrt[3]{x}} = x^{-2} \cdot x^{-1/3} = x^{-7/3} = \dfrac{1}{x^{7/3}}$

74. $\left[f(x)\right]^3 g(x) = \left(\sqrt[3]{x}\right)^3 \cdot \dfrac{1}{x^2} = x \cdot x^{-2} = x^{-1} = \dfrac{1}{x}$

75. $\left[f(x)g(x)\right]^3 = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^3 = \left(x^{1/3}\cdot x^{-2}\right)^3$
$= \left(x^{-5/3}\right)^3 = x^{-5} = \frac{1}{x^5}$

76. $\sqrt{\frac{f(x)}{g(x)}} = \left(\frac{\sqrt[3]{x}}{\frac{1}{x^2}}\right)^{1/2} = \left(x^{1/3}\cdot x^2\right)^{1/2} = \left(x^{7/3}\right)^{1/2}$
$= x^{7/6}$

77. $\sqrt{f(x)g(x)} = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^{1/2} = \left(x^{1/3}\cdot x^{-2}\right)^{1/2}$
$= \left(x^{-5/3}\right)^{1/2} = x^{-5/6} = \frac{1}{x^{5/6}}$

78. $\sqrt[3]{f(x)g(x)} = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^{1/3} = \left(x^{1/3}\cdot x^{-2}\right)^{1/3}$
$= \left(x^{-5/3}\right)^{1/3} = x^{-5/9} = \frac{1}{x^{5/9}}$

79. $f(g(x)) = f\left(\frac{1}{x^2}\right) = f\left(x^{-2}\right) = \sqrt[3]{x^{-2}}$
$= \left(x^{-2}\right)^{1/3} = x^{-2/3} = \frac{1}{x^{2/3}}$

80. $g(f(x)) = g\left(\sqrt[3]{x}\right) = g\left(x^{1/3}\right) = \left(\frac{1}{x^{1/3}}\right)^2 = \frac{1}{x^{2/3}}$

81. $f(g(x)) = f\left(\sqrt[3]{x}\right) = f\left(x^{1/3}\right) = \sqrt[3]{x^{1/3}}$
$= \left(x^{1/3}\right)^{1/3} = x^{1/9}$

82. $g(g(x)) = g\left(\frac{1}{x^2}\right) = g\left(x^{-2}\right) = \left(\frac{1}{x^{-2}}\right)^2$
$= \frac{1}{x^{-4}} = x^4$

83. $\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}(x-1)$

84. $2x^{2/3} - x^{-1/3} = x^{-1/3}(2x-1)$

85. $x^{-1/4} + 6x^{1/4} = x^{-1/4}\left(1+6\sqrt{x}\right)$

86. $\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \sqrt{xy}\left(\frac{1}{y} - \frac{1}{x}\right)$

87. $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$
$a^{1/2}\cdot b^{1/2} = (ab)^{1/2}$ (Law 5)

88. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
$\frac{a^{1/2}}{b^{1/2}} = \left(\frac{a}{b}\right)^{1/2}$ (Law 6)

89. $f(x) = x^2 \Rightarrow f(4) = (4)^2 = 16$

90. $f(x) = x^3 \Rightarrow f(4) = (4)^3 = 64$

91. $f(x) = x^{-1} \Rightarrow f(4) = (4)^{-1} = \frac{1}{4}$

92. $f(x) = x^{1/2} \Rightarrow f(4) = (4)^{1/2} = 2$

93. $f(x) = x^{3/2} \Rightarrow f(4) = (4)^{3/2} = 8$

94. $f(x) = x^{-1/2} \Rightarrow f(4) = (4)^{-1/2} = \frac{1}{2}$

95. $f(x) = x^{-5/2} \Rightarrow f(4) = (4)^{-5/2} = \frac{1}{32}$

96. $f(x) = x^0 \Rightarrow f(4) = 4^0 = 1$

In exercises 97−104, use the compound interest formula $A = P\left(1+\frac{r}{m}\right)^{mt}$, where P is the principal, r is the annual interest rate, m is the number of interest periods per year, and t is the number of years.

97. $A = 500\left(1+\frac{.06}{1}\right)^{1(6)} \approx \$709.26$

98. $A = 700\left(1+\frac{.08}{1}\right)^{1(8)} \approx \$1295.65$

99. $A = 50,000\left(1+\frac{.095}{4}\right)^{4(10)} \approx \$127,857.61$

100. $A = 20,000\left(1+\frac{.12}{4}\right)^{4(3)} \approx \$28,515.22$

101. $A = 100\left(1+\frac{.05}{12}\right)^{12(10)} \approx \$164.70$

102. $A = 500\left(1+\frac{.045}{12}\right)^{12(1)} \approx \$522.97$

103. $A = 1500\left(1+\frac{.06}{365}\right)^{365(1)} \approx \$1592.75$

104. $A = 1500\left(1 + \frac{.06}{365}\right)^{365(3)} \approx \$1795.80$

105. $A = 1000\left(1 + \frac{.068}{1}\right)^{1(18)} \approx \$3268.00$

106. At the end of the first year, there will be $A_1 = A_0(1 + .08) = 4000(1.08) = \$4320$ in the account. At the end of the second year, there will be

$$A_2 = A_1(1 + .08) = (4320 + 4000)(1.08) = \$8985.60$$

in the account. At the end of the third year, there will be

$$A_3 = A_2(1 + 0.8) = (8985.60 + 4000)(1.08) = 14{,}024.448$$

in the account. (Note that we hold the decimals since this is a partial answer. We will round at the end of the calculations.) At the end of the fourth year, there will be

$$A_4 = A_3(1 + .08) = (14{,}024.448 + 4000)(1.08) \approx 19{,}466.40384$$

in the account. No additional deposits are made, so use the compound interest formula to compute the amount in the account after another four years:

$$A = 19{,}466.40384\left(1 + \frac{.08}{1}\right)^{1(4)} \approx \$26{,}483.83.$$

107. $A = 500 + 500r + \frac{375}{2}r^2 + \frac{125}{4}r^3 + \frac{125}{64}r^4 = \frac{500}{256}\left(256 + 256r + 96r^2 + 16r^3 + r^4\right)$

108. $A = 1000 + 2000r + 1500r^2 + 500r^3 + \frac{125}{2}r^4 = \frac{125}{2}\left(16 + 32r + 24r^2 + 8r^3 + r^4\right)$

109. If the speed is $2x$, then $\frac{1}{20}(2x)^2 = \frac{1}{20}\left(4x^2\right) = 4\left(\frac{1}{20}x^2\right).$

110. 5E–5 $= 5 \cdot 10^{-5} = .00005$

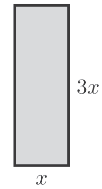
111. 8.103E–4 $= 8.103 \cdot 10^{-4} = .0008103$

112. 1.35E13 $= 1.35 \cdot 10^{13} = 13{,}500{,}000{,}000{,}000$

113. 8.23E–6 $= 8.23 \cdot 10^{-6} = .00000823$

0.6 Functions and Graphs in Applications

1.



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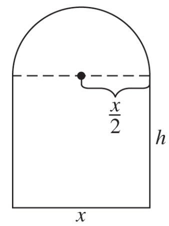

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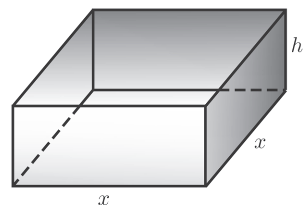

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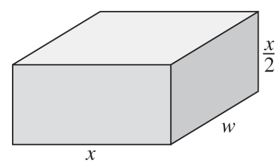

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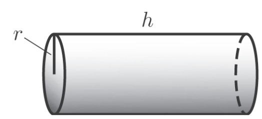
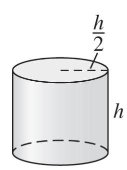


6.



7. $P = 2(x + 3x) = 8x$

$3x^2 = 25$

8. $A = 3x^2$

$8x = 30$

9. $A = \pi r^2$

$2\pi r = 15$

10. $P = 2r + 2h + \pi r$

The area of the window is represented by $A = 2rh + \frac{1}{2}\pi r^2.$

$2rh + \left(\frac{1}{2}\right)\pi r^2 = 2.5$

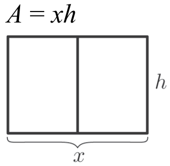
11. $V = x^2h$
The surface area of the box is represented by
$S = x^2 + 4xh.$
$x^2 + 4xh = 65$

12. $\text{SA} = 2xw + 2x\left(\frac{x}{2}\right) + 2w\left(\frac{x}{2}\right) = 3xw + x^2$
The volume is represented by
$xw\left(\frac{x}{2}\right) = \frac{1}{2}x^2w.$
$\left(\frac{1}{2}\right)wx^2 = 10$

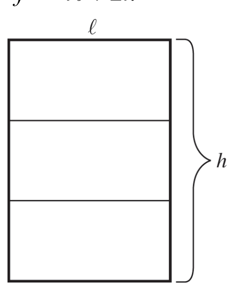
13. $\pi r^2h = 100$
$\text{Cost} = 5\pi r^2 + 6\pi r^2 + 7(2\pi rh)$
$= 11\pi r^2 + 14\pi rh$

14. $2\pi\left(\frac{h}{2}\right)^2 + 2\pi\left(\frac{h}{2}\right)h = \frac{\pi h^2}{2} + \pi h^2$
$= \frac{3\pi h^2}{2} = 30\pi$
$V = \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{4}$

15. $2x + 3h = 5000$
$A = xh$



16. $\ell h = 2500$
$f = 4\ell + 2h$



17. $C = 10(2\ell + 2h) + 8(2\ell) = 36\ell + 20h$

18. $5x^2 + 4(4xh) = 5x^2 + 16xh = 150$

19. $8x = 40 \Rightarrow x = 5$
$A = 3x^2 = 3(25) = 75 \text{ cm}^2$

20. $V = 2\pi r^3 = 54\pi \Rightarrow r^3 = 27 \Rightarrow r = 3$
From exercise 14, we know that the surface area is equal to $6\pi r^2$. Thus, in this example $S = 6\pi\left(3^2\right) = 54\pi \text{ in.}^2$

21. a. $73 + 4x = 225 \Rightarrow x = 38$
When 38 T-shirts are sold, the cost will be \$225.

b. $C(50) - C(40)$
$= \left(73 + 4(50)\right) - \left(73 + 4(40)\right)$
$= 273 - 233 = \$40$
The cost will rise \$40.

22. a. $P(x) = 4x - C(x)$
$P(100) = 400 - (10 + 75) = \$315$

b. $P(101) = 404 - (10.1 + 75) = \$318.9$
Increase is \$3.90.

23. a. $.4x - 80 = 0 \Rightarrow x = \frac{80}{.4} = 200$
Sales will break-even when 200 scoops are sold.

b. $30 = .4x - 80 \Rightarrow x = 275$
Sales of 275 scoops will generate a daily profit of \$30.

c. $40 = .4x - 80 \Rightarrow x = 300$
To raise the daily profit to \$40, 300 – 275 = 25 more scoops will have to be sold.

24. a. $160 = 12x - 200 \Rightarrow x = 30$
30 thousand subscribers are needed for a monthly profit of \$160 thousand

b. $166 = 12x - 200 \Rightarrow x = 30.5$ thousand
There will need to be 30,500 – 30,000 = 500 new subscribers.

25. a. $P(x) = R(x) - C(x) = 21x - 9x - 800$
$= 12x - 800$

b. $P(120) = 1440 - 800 = \$640$

c. $1000 = 12x - 800 \Rightarrow x = 150$
$R(150) = 21(150) = \$3150$

26. a. $P(x) = R(x) - C(x)$
$= 1200x - (550x + 6500)$
$= 650x - 6500$
$P(12) = 650(12) - 6500 = \$1300$
The company will earn \$1300.

b. $C(x) = 14,750 = 550x + 6500 \Rightarrow x = 15$
$P(15) = 650(15) - 6500 = \$3250$

27. $f(6) = 270$ cents

28. From the graph, $f(r) = 330$ for $r = 1$ and $r = 6.87$.

29. A 100-inch^3 cylinder with radius 3 inches costs \$1.62 to construct.

30. The least expensive cylinder has radius 3 inches and costs \$1.62 to construct.
The cost drops until the radius is 3 in. and then increases.

31. $f(3) = \$1.62$; $f(6) = \$2.70$, so the additional cost $= 2.70 - 1.62 = \$1.08$

32. $f(1) = \$3.30$; $f(3) = \$1.62$, so the amount saved is $3.30 - 1.62 = \$1.68$

33. From the graph, we see that revenue = \$1800 and cost = \$1200.

34. The revenue is \$1400 when production is 20 units.

35. The cost is \$1400 when production is 40 units.

36. $1800 - 1200 = \$600$

37. $C(1000) = \$4000$

38. Find the x-coordinate of the point on the graph whose y-coordinate is 3500.

39. Find the y-coordinate of the point on the graph whose x-coordinate is 400.

40. $C(600) - C(500) = 3136 - 2875 = \$261$

41. The greatest profit, \$52,500, occurs when 2500 units of goods are produced.

42. $P(1500) = \$42{,}500$

43. Find the x-coordinate of the point on the graph whose y-coordinate is 30,000.

44. Find the y-coordinate of the point on the graph whose x-coordinate is 2000.

45. Find $h(3)$. Find the y-coordinate of the point on the graph whose t-coordinate is 3.

46. Find t such that $h(t)$ is as large as possible. Find the t-coordinate of the highest point of the graph.

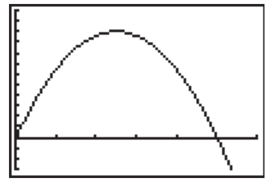
47. Find the maximum value of $h(t)$. Find the y-coordinate of the highest point of the graph.

48. Solve $h(t) = 0$. Find the t-intercept of the graph.

49. Solve $h(t) = 100$. Find the t-coordinates of the points whose y-coordinate is 100.

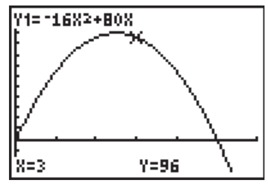
50. Find $h(0)$. Find the y-intercept of the graph.

51. a.

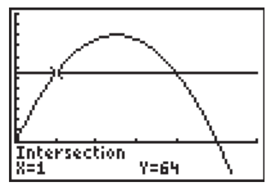


[0, 6] by [−30, 120]

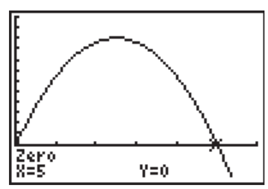
b. Using the Trace command or the Value command, the height is 96 feet.



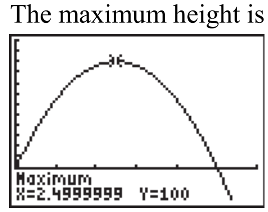
c. Graphing $Y_2 = 64$ and using the Intersect command, the height is 64 feet when $x = 1$ and $x = 4$ seconds.



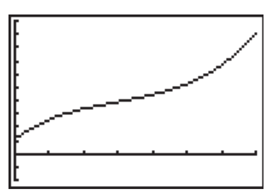
d. Using the Trace command or the Zero command, the ball hits the ground when $x = 5$ seconds.



e. Using the Trace command or the Maximum command, the maximum height is reached when $x = 2.5$ seconds. The maximum height is 100 feet.

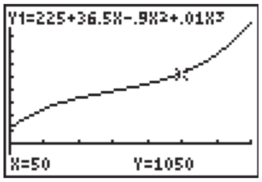


52. a.

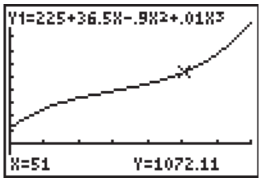


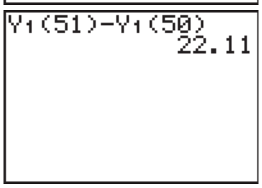
[0, 70] by [−400, 2000]

b. Using the Trace command or the Value command, the cost is \$1050.



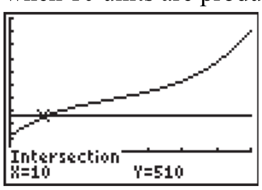
c.



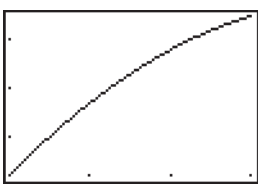


The additional cost is \$22.11.

d. Graphing $Y_2 = 510$ and using the Intersect command, the daily cost is \$510 when 10 units are produced.

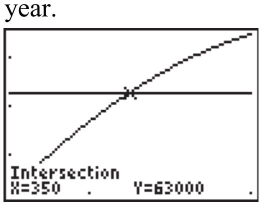


53. a.

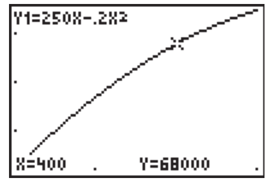


[200, 500] by [42000, 75000]

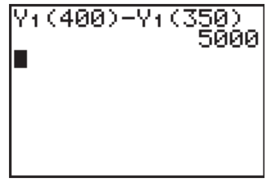
b. Graphing $Y_2 = 63,000$ and using the Intersect command, the revenue is \$63,000 when sales are 350 bicycles per year.



c. Using the Trace command or the Value command, the revenue is \$68,000 when 400 bicycles are sold per year.

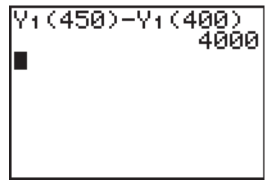


d.



$R(400) - R(350) = 5000$
Revenue would decrease by \$5000.

e.



$R(450) - R(400) = 4000$
No, the store should not spend \$5000 on advertising, since the revenues would only increase by \$4000.

Chapter 0 Fundamental Concept Check Exercises

1. Real numbers can be thought of as points on a number line, where each number corresponds to one point on the line, and each point determines one real number. Every real number has a decimal representation. A rational number is a real number with a finite or infinite repeating decimal, such as $-\frac{5}{2} = -2.5,\ 1,\ \frac{13}{3} = 4.33\overline{3}$. An irrational number is a real number with an infinite, non-repeating decimal representation, such as $-\sqrt{2} = -1.414213\ldots$ or $\pi = 3.14159\ldots$.

2. $x < y$ means x is less than y; $x \le y$ means x is less than or equal to y; $x > y$ means x is greater than y; $x \ge y$ means x is greater than or equal to y.

3. An open interval (a, b) does not contain its endpoints a and b but a closed interval $[a, b]$ does not contain a and b.

4. A function of a variable x is a rule f that assigns a unique number $f(x)$ to each value of x.

5. The value of a function at x is the unique number $f(x)$.

6. The domain of a function is the set of values that the independent variable x is allowed to assume. The range of a function is the set of values that the function assumes.

7. The graph of a function $f(x)$ is the curve that consists of the set of all points $(x, f(x))$ in the xy-plane. A curve is the graph of a function if and only if each vertical line cuts or touches the curve at no more than one point.

8. A linear function has the form $f(x) = mx + b$. When $m = 0$, the function is a constant function. $f(x) = 3x - .5$ is a linear function. $f = -2$ is a constant function.

9. An x-intercept is a point at which the graph of a function intersects the x-axis. A y-intercept is a point at which the graph intersects the y-axis. To find the x-intercept, set $f(x) = 0$ and solve for x, if possible. The y-intercept is the point $(0, f(0))$.

10. A quadratic function has the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph is a parabola.

11. **a.** Quadratic function: $f(x) = ax^2 + bx + c$, where $a \neq 0$; $f(x) = -2x^2 + 4x + 9$

b. Polynomial function: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers, $a_n \neq 0$, and n is a nonnegative integer; $f(x) = x^5 + 3x^3 - 7x + 3$

c. Rational function: $h(x) = \dfrac{f(x)}{g(x)}$, where f and g are polynomials; $h(x) = \dfrac{2x-3}{x^2+1}$

d. Power function: $f(x) = x^r$, where r is a real number; $f(x) = \sqrt{x} = x^{1/2}$

12. $f(x) = |x|$ is defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$$

13. Sum: $f(x) + g(x)$

Difference: $f(x) - g(x)$

Product: $f(x)g(x)$

Quotient: $\dfrac{f(x)}{g(x)}$

Composition: $f(g(x))$

If $f(x) = 3x^2$ and $g(x) = 3x + 1$, then

$$f(x) + g(x) = 3x^2 + 3x + 1$$
$$f(x) - g(x) = 3x^2 - (3x + 1) = 3x^2 - 3x - 1$$
$$f(x)g(x) = 3x^2(3x + 1) = 9x^3 + 3x^2$$
$$\frac{f(x)}{g(x)} = \frac{3x^2}{3x+1}$$
$$f(g(x)) = 3(3x+1)^2 = 3(9x^2 + 6x + 1) = 27x^2 + 18x + 3$$

14. $x = a$ is a zero of $f(x)$ if $f(a) = 0$.

15. Two methods for finding the zeros of a quadratic function are using factoring or using the quadratic equation.

16. $b^r b^s = b^{r+s}$ $\quad b^{-r} = \dfrac{1}{b^r}$

$\dfrac{b^r}{b^s} = b^{r-s}$ $\quad (b^r)^s = b^{rs}$

$(ab)^r = a^r b^r$ $\quad \left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$

17. In the formula $A = P(1+i)^n$, A represents the compound amount, P represents the principal amount, i represents the interest rate, and n represents the number of interest periods.

18. To solve $f(x) = b$ geometrically from the graph of $y = f(x)$, draw the horizontal line $y = b$. The line intersects the graph at a point (a, b) if and only if $f(a) = b$. Thus, $x = a$ is a solution of $f(x) = b$.

19. To find $f(a)$ geometrically from the graph of $y = f(x)$, draw the vertical line $x = a$. This line intersects the graph at the point $(a, f(a))$.

Chapter 0 Review Exercises

1. $f(x) = x^3 + \frac{1}{x}$

$f(1) = 1^3 + \frac{1}{1} = 2$

$f(3) = 3^3 + \frac{1}{3} = \frac{82}{3} = 27\frac{1}{3}$

$f(-1) = (-1)^3 + \frac{1}{(-1)} = -2$

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 2 = -\frac{17}{8} = -2\frac{1}{8}$

$f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^3 + \frac{1}{\sqrt{2}} = 2\sqrt{2} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

2. $f(x) = 2x + 3x^2$

$f(0) = 2(0) + 3(0)^2 = 0$

$f\left(-\frac{1}{4}\right) = 2\left(-\frac{1}{4}\right) + 3\left(-\frac{1}{4}\right)^2 = -\frac{5}{16}$

$f\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3 + 2\sqrt{2}}{2}$

3. $f(x) = x^2 - 2$

$f(a-2) = (a-2)^2 - 2 = a^2 - 4a + 2$

4. $f(x) = \frac{1}{x+1} - x^2$

$$\begin{aligned} f(a+1) &= \frac{1}{(a+1)+1} - (a+1)^2 \\ &= \frac{1}{a+2} - (a^2 + 2a + 1) \\ &= -\frac{a^3 + 4a^2 + 5a + 1}{a+2} \end{aligned}$$

5. $f(x) = \frac{1}{x(x+3)} \Rightarrow x \neq 0, -3$

6. $f(x) = \sqrt{x-1} \Rightarrow x \geq 1$

7. $f(x) = \sqrt{x^2 + 1}$, all values of x

8. $f(x) = \frac{1}{\sqrt{3x}}$, $x > 0$

9. $h(x) = \frac{x^2 - 1}{x^2 + 1}$

$h\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + 1} = -\frac{3}{5}$

Yes, the point $\left(\frac{1}{2}, -\frac{3}{5}\right)$ is on the graph.

10. $k(x) = x^2 + \frac{2}{x}$

$k(1) = 1^2 + \frac{2}{1} = 3$

No, the point (1, –2) is not on the graph.

11. $$\begin{aligned} 5x^3 + 15x^2 - 20x &= 5x(x^2 + 3x - 4) \\ &= 5x(x-1)(x+4) \end{aligned}$$

12. $$\begin{aligned} 3x^2 - 3x - 60 &= 3(x^2 - x - 20) \\ &= 3(x-5)(x+4) \end{aligned}$$

13. $$\begin{aligned} 18 + 3x - x^2 &= (-x-3)(x-6) \\ &= (-1)(x-6)(x+3) \end{aligned}$$

14. $$\begin{aligned} x^5 - x^4 - 2x^3 &= x^3(x^2 - x - 2) \\ &= x^3(x-2)(x+1) \end{aligned}$$

15. $y = 5x^2 - 3x - 2 \Rightarrow 5x^2 - 3x - 2 = 0.$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-2)}}{2(5)} \\ &= \frac{3 \pm 7}{10} \Rightarrow x = 1 \text{ or } x = -\frac{2}{5} \end{aligned}$$

16. $y = -2x^2 - x + 2 \Rightarrow -2x^2 - x + 2 = 0.$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4(-2)(2)}}{2(-2)} \\ &= \frac{1 \pm \sqrt{17}}{-4} \Rightarrow x = \frac{-1 + \sqrt{17}}{4} \text{ or } x = \frac{-1 - \sqrt{17}}{4} \end{aligned}$$

17. Substitute $2x - 1$ for y in the quadratic equation, then find the zeros:

$$5x^2 - 3x - 2 = 2x - 1 \Rightarrow 5x^2 - 5x - 1 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4(5)(-1)}}{2(5)} = \frac{5 \pm 3\sqrt{5}}{10}$$

Now find the y-values for each x value:

$$y = 2x - 1 = 2\left(\frac{5 + 3\sqrt{5}}{10}\right) - 1 = \frac{3\sqrt{5}}{5}$$

$$y = 2x - 1 = 2\left(\frac{5 - 3\sqrt{5}}{10}\right) - 1 = \frac{-3\sqrt{5}}{5}$$

Points of intersection:

$$\left(\frac{5 + 3\sqrt{5}}{10}, \frac{3\sqrt{5}}{5}\right), \left(\frac{5 - 3\sqrt{5}}{10}, -\frac{3\sqrt{5}}{5}\right)$$

18. Substitute $x - 5$ for y in the quadratic equation, then find the zeros:

$$-x^2 + x + 1 = x - 5 \Rightarrow x^2 - 6 = 0 \Rightarrow x = \pm\sqrt{6}$$

Now find the y-values for each x value:

$$y = x - 5 = \sqrt{6} - 5$$

$$y = -\sqrt{6} - 5$$

Points of intersection:

$$\left(\sqrt{6}, \sqrt{6} - 5\right), \left(-\sqrt{6}, -\sqrt{6} - 5\right)$$

19. $f(x) + g(x) = \left(x^2 - 2x\right) + \left(3x - 1\right) = x^2 + x - 1$

20.
$$\begin{aligned} f(x) - g(x) &= \left(x^2 - 2x\right) - \left(3x - 1\right) \\ &= x^2 - 5x + 1 \end{aligned}$$

21.
$$\begin{aligned} f(x)h(x) &= \left(x^2 - 2x\right)\left(\sqrt{x}\right) \\ &= x^2 \cdot x^{1/2} - 2x \cdot x^{1/2} \\ &= x^{5/2} - 2x^{3/2} \end{aligned}$$

22.
$$\begin{aligned} f(x)g(x) &= (x^2 - 2x)(3x - 1) \\ &= 3x^3 - x^2 - 6x^2 + 2x \\ &= 3x^3 - 7x^2 + 2x \end{aligned}$$

23. $\dfrac{f(x)}{h(x)} = \dfrac{x^2 - 2x}{\sqrt{x}} = x^{3/2} - 2x^{1/2}$

24.
$$\begin{aligned} g(x)h(x) &= (3x - 1)\sqrt{x} = 3x \cdot x^{1/2} - x^{1/2} \\ &= 3x^{3/2} - x^{1/2} \end{aligned}$$

25.
$$\begin{aligned} f(x) - g(x) &= \frac{x}{x^2 - 1} - \frac{1 - x}{1 + x} \\ &= \frac{x - (x - 1)(1 - x)}{x^2 - 1} \\ &= \frac{x^2 - x + 1}{x^2 - 1} = \frac{x^2 - x + 1}{(x - 1)(x + 1)} \end{aligned}$$

26.
$$\begin{aligned} f(x) - g(x + 1) &= \frac{x}{x^2 - 1} - \frac{1 - (x + 1)}{1 + (x + 1)} \\ &= \frac{x(x + 2) - (-x)\left(x^2 - 1\right)}{\left(x^2 - 1\right)(x + 2)} \\ &= \frac{x^3 + x^2 + x}{\left(x^2 - 1\right)(x + 2)} \end{aligned}$$

27.
$$\begin{aligned} g(x) - h(x) &= \frac{1 - x}{1 + x} - \frac{2}{3x + 1} \\ &= \frac{(1 - x)(3x + 1) - 2(1 + x)}{(1 + x)(3x + 1)} \\ &= -\frac{3x^2 + 1}{(1 + x)(3x + 1)} \\ &= -\frac{3x^2 + 1}{3x^2 + 4x + 1} \end{aligned}$$

28.
$$\begin{aligned} f(x) + h(x) &= \frac{x}{x^2 - 1} + \frac{2}{3x + 1} \\ &= \frac{x(3x + 1) + 2\left(x^2 - 1\right)}{\left(x^2 - 1\right)(3x + 1)} \\ &= \frac{5x^2 + x - 2}{\left(x^2 - 1\right)(3x + 1)} \end{aligned}$$

29.
$$\begin{aligned} g(x) - h(x - 3) &= \frac{1 - x}{1 + x} - \frac{2}{3(x - 3) + 1} \\ &= \frac{(1 - x)(3x - 8) - 2(1 + x)}{(1 + x)(3x - 8)} \\ &= \frac{-3x^2 + 9x - 10}{(1 + x)(3x - 8)} \\ &= \frac{-3x^2 + 9x - 10}{3x^2 - 5x - 8} \end{aligned}$$

30.
$$\begin{aligned} f(x) + g(x) &= \frac{x}{x^2 - 1} + \frac{1 - x}{1 + x} = \frac{x + (1 - x)(x - 1)}{x^2 - 1} \\ &= \frac{-x^2 + 3x - 1}{x^2 - 1} \end{aligned}$$

For exercises 31–36, $f(x)=x^2-2x+4$, $g(x)=\frac{1}{x^2}$ and $h(x)=\frac{1}{\sqrt{x}-1}$.

31. $f(g(x))=f\left(\frac{1}{x^2}\right)=\left(\frac{1}{x^2}\right)^2-2\left(\frac{1}{x^2}\right)+4$
$=\frac{1}{x^4}-\frac{2}{x^2}+4$

32. $g(f(x))=g(x^2-2x+4)=\frac{1}{\left(x^2-2x+4\right)^2}$
$=\frac{1}{x^4-4x^3+12x^2-16x+16}$

33. $g(h(x))=g\left(\frac{1}{\sqrt{x}-1}\right)=\frac{1}{\left(\frac{1}{\sqrt{x}-1}\right)^2}=\frac{1}{\frac{1}{x-2\sqrt{x}+1}}$
$=x-2\sqrt{x}+1=\left(\sqrt{x}-1\right)^2$

34. $h(g(x))=h\left(\frac{1}{x^2}\right)=\frac{1}{\sqrt{\frac{1}{x^2}}-1}=\frac{1}{\frac{1}{|x|}-1}=\frac{|x|}{1-|x|}$

35. $f(h(x))=f\left(\frac{1}{\sqrt{x}-1}\right)$
$=\left(\frac{1}{\sqrt{x}-1}\right)^2-2\left(\frac{1}{\sqrt{x}-1}\right)+4$
$=\frac{1}{\left(\sqrt{x}-1\right)^2}-\frac{2}{\sqrt{x}-1}+4$

36. $h(f(x))=h\left(x^2-2x+4\right)$
$=\frac{1}{\sqrt{x^2-2x+4}-1}$
$=\left(\sqrt{x^2-2x+4}-1\right)^{-1}$

37. $(81)^{3/4}=\left(\sqrt[4]{81}\right)^3=27$
$8^{5/3}=\left(\sqrt[3]{8}\right)^5=2^5=32$
$(0.25)^{-1}=\left(\frac{1}{4}\right)^{-1}=4$

38. $(100)^{3/2}=\left(\sqrt{100}\right)^3=1000$
$(.001)^{1/3}=\left(\sqrt[3]{.001}\right)=.1$

39. $C(x)$ = carbon monoxide level corresponding to population x
$P(t)$ = population of the city in t years
$C(x)=1+.4x$
$P(t)=750+25t+.1t^2$
$C(P(t))=1+.4\left(750+25t+.1t^2\right)$
$=1+300+10t+.04t^2$
$=.04t^2+10t+301$

40. $R(x)=5x-x^2$
$f(d)=6\left(1-\frac{200}{d+200}\right)$
$R(f(d))=5\cdot 6\left(1-\frac{200}{d+200}\right)$
$-\left[6\left(1-\frac{200}{d+200}\right)\right]^2$
$=30\left(1-\frac{200}{d+200}\right)-36\left(1-\frac{200}{d+200}\right)^2$

41. $\left(\sqrt{x+1}\right)^4=(x+1)^{4/2}=(x+1)^2=x^2+2x+1$

42. $\frac{xy^3}{x^{-5}y^6}=x\cdot x^5\cdot y^3\cdot y^{-6}=\frac{x^6}{y^3}$

43. $\frac{x^{3/2}}{\sqrt{x}}=x^{3/2}\cdot x^{-1/2}=x$

44. $\sqrt[3]{x}(8x^{2/3})=x^{1/3}\cdot 8x^{2/3}=8x$

45. a. $P=15000,\ r=.04,\ m=12$
$A(t)=15000\left(1+\frac{.04}{12}\right)^{12t}$
$=15000(1.00333)^{12t}$

b. $A(2)=15000\left(1+\frac{.04}{12}\right)^{12\cdot 2}\approx 16247.14$
$A(5)=15000\left(1+\frac{.04}{12}\right)^{12\cdot 5}\approx 18314.94$
At the end of 2 years, the account balance is about \$16,247. At the end of 5 years, the account balance is about \$18,315.

46. a. $P=7000,\ r=.09,\ m=2$
$A(t)=7000\left(1+\frac{.09}{2}\right)^{2t}=7000(1.045)^{2t}$

b. $A(10) = 7000(1.045)^{2\cdot 10} = 16882.00$
$A(20) = 7000(1.045)^{2\cdot 20} = 40714.55$
At the end of 10 years, the account balance is about \$16,882. At the end of 20 years, the account balance is about \$40,715.

47. a. $P = 15000,\ m = 1,\ t = 10$
$A(r) = 15000(1+r)^{10}$

b. $A(.04) = 15000(1+.04)^{10} = 22203.66$
$A(.06) = 15000(1+.06)^{10} = 26862.72$

48. a. $P = 7000,\ m = 1,\ t = 20$
$A(r) = 7000(1+r)^{20}$

b. $A(.07) = 7000(1+.07)^{20} = 27087.79$
$A(.12) = 7000(1+.12)^{20} = 67524.05$